Charles L. Keller and Mark C. Breiter, Bounds on the value of a Dirichlet integral representing a coefficient of apparent mass

Paruchuri R. Krishnaiah, Selection procedures based on covariance matrices of multivariate normal populations

Yudell L. Luke, Recursion formulas for polynomials in rational approximations to generalized hypergeometric functions

Mary D. Lum, On degradation of combination locks and the maximum time to open them

Robert E. Lynch, Generalized trapezoid formulas and errors in Romberg quadrature

Knox Millsaps and Norman L. Soong, Heat transfer in a plane incompressible laminar jet

Bertram Mond and Donald S. Clemm, Some computational aspects of the tensor product of linear programs

Bertram Mond and Oved Shisha, A difference inequality for operators in Hilbert space

Ida Rhodes, The mighty man-computer team

Paul R. Rider, Products and quotients of generalized Cauchy variables

Max G. Scherberg, Explorations of supersonic shear flow over a cavity

Henry C. Thacher, Jr., Computation of the complex error function by continued fractions

Peter Wynn, Transformations to accelerate the convergence of Fourier series.

It might have been fitting, for this occasion, to include a scientific biography of Dr. Blanch and a list of her publications. Even so, Dr. Blanch's own interests, influence, and guidance, are reflected in a good many of the papers presented.

W. G.

 82[3, 4, 5, 13.15, 13.35].—DONALD GREENSPAN, Editor, Numerical Solutions of Nonlinear Differential Equations, John Wiley & Sons, Inc., New York, 1966, x + 347 pp., 24 cm. Price \$7.75.

These are the proceedings of an Advanced Symposium conducted by the Mathematics Research Center, U. S. Army, at the University of Wisconsin, May 9–11, 1966, containing fourteen lectures by invited speakers, and 24 abstracts of contributed papers.

The principal authors and their titles are as follows:

Garrett Birkhoff, Numerical solution of the reactor kinetics equations

J. R. Cannon, Some numerical results for the solution of the heat equation backwards in time

C. W. Clenshaw, The solution of van der Pol's equation in Chebyshev series

L. Collatz, Monotonicity and related methods in non-linear differential equations problems

Germund G. Dahlquist, On rigorous error bounds in the numerical solution of ordinary differential equations

Stuart E. Dreyfus, The numerical solution of non-linear optimal control problems

H. B. Keller and H. Takami, Numerical studies of steady viscous flow about cylinders

Heinz-Otto Kreiss, Difference approximations for the initial-boundary value problem for hyperbolic differential equations

M. Lees and M. H. Schultz, A Leray-Schauder principle for A-compact mappings and the numerical solution of non-linear two-point boundary value problems

William F. Noh, A general theory for the numerical solution of the equations of hydrodynamics

Seymour V. Parter, Maximal solutions of mildly non-linear elliptic equations

L. E. Payne, On some non-well-posed problems for partial differential equations

J. B. Rosen, Approximate computational solution of non-linear parabolic partial differential equations by linear programming

Minoru Urabe, Galerkin's procedure for non-linear periodic systems and its extension to multi-point boundary value problems for general non-linear systems.

Birkhoff proposes a combination of singular perturbation and numerical integration for dealing with the notoriously delicate reactor kinetics equations. Cannon reports on some numerical experiments of continuing a solution of the heat equation backwards in time. Clenshaw describes a doubly iterative scheme for computing the periodic solution of the van der Pol equation to high accuracy, noting certain advantages of economy resulting from the use of Chebyshev series. Collatz gives many examples to show how ideas from functional analysis, notably monotonicity principles and fixed-point theorems, can fruitfully be employed to obtain error bounds for the approximate solution of complicated nonlinear problems. Dahlquist explores the use of majorant functions to construct rigorous error bounds in the numerical solution of ordinary differential equations. Dreyfus discusses the solution of general nonlinear optimal control problems using secondorder variational analysis and dynamic programming. Keller and Takami present detailed results for the numerical solution of the Navier-Stokes equations. They consider steady two-dimensional incompressible viscous flow about a circular cylinder and report on successful calculations for Reynold numbers as high as 15. Kreiss investigates  $L_2$ -stability of difference approximations to hyperbolic partial differential equations with variable coefficients. Lees and Schultz formulate two fixed-point theorems for "approximately compact" mappings, analogous to the Leray-Schauder principle for completely continuous mappings, and apply them to obtain existence and convergence theorems for the solution of two-point boundary value problems by finite difference methods. Noh discusses difference schemes for the numerical solution of flow problems involving contact discontinuities. Striking numerical results are presented for the case of a hyper-velocity pellet impacting on a target. Parter examines the stability, and approximation by difference methods, of maximal solutions to nonlinear boundary value problems admitting more than one solution. Payne discusses stabilization procedures for solutions of non-wellposed problems for second order systems of partial differential equations. Rosen investigates the application of linear programming to the approximate solution of nonlinear parabolic differential equations and reports on the results for a number of test problems. Urabe studies the solution of multipoint boundary value problems for systems of nonlinear ordinary differential equations by Galerkin's method using finite Fourier and Chebyshev series. In particular, he gives convergence theorems, and conditions under which the existence of a solution can be ascertained from computed approximate solutions.

Rapid progress in computer technology, in recent years, has made it possible to

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attack nonlinear differential equations problems of great complexity, and thus has led to a renewed interest in the design and analysis of relevant computational methods. The present volume provides a well-balanced, and well-documented, survey of current research in this area.

W. G.

## 83[4, 6].—LOUIS BRAND, Differential and Difference Equations, John Wiley & Sons, Inc., New York, 1966, xvi + 698 pp., 24 cm. Price \$11.95.

This is a skillfully written introductory book on the more classical parts of the subject. Its distinguishing features are the conscious effort made to bring out the formal analogies and interplays between differential and difference equations, the generous consideration given to applications in the physical sciences, and the inclusion of Mikusiński's operational calculus, both for functions of a continuous and a discrete variable. The chapter headings are: 1. Differential equations of the first order, 2. Important types of first-order equations, 3. Linear equations of the second order, 4. Linear equations with constant coefficients, 5. Systems of equations, 6. Applications, 7. Laplace transform, 8. Linear difference equations, 9. Linear difference equations with constant coefficients, 10. Solutions in series, 11. Mikusiński's operational calculus, 12. Existence and uniqueness theorems, 13. Interpolation and numerical integration, 14. Numerical methods.

W. G.

84[7].—ALDO ASCARI, A Table of the Repeated Integrals of the Error Function, Internal Report S/6339, Società Richerche Impianti Nucleari (SORIN), Nuclear Research Center, Saluggia, Vercelli, Italy, 1968, v + 104 pp., 31 cm. Deposited in UMT file.

This table consists of 10S unrounded values (in floating-point form) of the normalized repeated integrals,  $A_n i^n$  erfc x, of the complementary error function  $2\pi^{-1/2} \int_x^{\infty} e^{-t^2} dt$ , for the range n = 1(1)24, x = 0(0.01)5.20.

The corresponding values of  $A_n = 2^n \Gamma((n/2) + 1)$  and its reciprocal are listed to 12S (also unrounded) in a preliminary table.

We are informed in the introduction that this table evolved as a by-product of the numerical solution of a specific diffusion problem, obtained on an Olivetti-Elea 6001/S computer at the Center.

The computation of the successive integrals was performed by means of the standard three-term recurrence formula, which was decomposed into a system of two first-order recurrences. The author states that several recurrent checks were applied to random entries, and were found invariably to be satisfied to at least 9S.

At the conclusion of the introductory remarks it is stated that the tabular entries were all obtained by chopping the 14S computer results to 10S. This, of course, means that terminal-digit errors can range up to nearly a unit.

Included in the appended list of six references are the tables of Berlyand et al. [1], which the present author has found unreliable, confirming the evaluation thereof made by this reviewer. He also refers to the comparatively brief, but useful, table of Gautschi [2].

In the opinion of this reviewer, the present table supersedes all previous tables